

Cepheids as Distance Indicators: Some Current Problems

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ABSTRACT. A general review is given of the calibration of the Cepheid distance scale, with particular reference to its use in the determination of H_0 . Emphasis is placed on the advantage of using a galactic calibration of the Cepheid scale, rather than relying on an adopted distance to the LMC. It is then possible to use LMC data to test for possible metallicity effects on this scale.

Key words: Cepheids, Distance Scales, Magellanic Clouds, H_0 .

1. Introduction

Cepheid variables are at the present time of key importance for establishing distances within our own Galaxy and to nearby galaxies and as the basis for the determination of cosmological parameters. As is well known, the determination of H_0 by groups working with the Hubble Space Telescope (e.g. Saha et al. 1999, Freedman et al. 2001 = F2001) rests on Cepheid observations in relatively nearby galaxies which are then used as calibrators of more general distance indicators (e.g. SNIa, the Tully-Fisher relation, surface brightness fluctuations etc.) It is therefore important that every effort should be made to understand these stars and their luminosities, and particularly to determine where present uncertainties lie.

The requirements of a primary distance indicator are easily stated. They must be bright, so that they can be seen and measured at large distances. They must also be recognizable for what they are. That is, they must not be confused with other types of objects. Their absolute magnitudes must be accurately known. This means that the intrinsic scatter in the absolute magnitudes must be small and that these absolute magnitudes can be accurately calibrated locally. There are two other considerations which are of equal importance to these. Firstly, the reddenings must be measurable. This is a crucial requirement. Secondly if there are effects due, or likely to be due, to differing metallicities or ages between the calibrators and the programme stars, these effects must be known and measurable. In addition to all this, it is highly desirable that the calibration and use of a primary distance indicator should be as free as possible from theoretical derivations or

assumptions. Unless we can establish distance scales empirically we have no way of testing theory.

Very few objects, if any, can match the classical Cepheids in fulfilling the requirements just set out. The main reason for this is that they have been extensively studied both locally and in the Magellanic Clouds for many years by a large number of investigators. It can thus be fairly claimed that they are the best understood of all pulsating variables. Other distance indicators will be mentioned below, but it can also be claimed with justification that Cepheids are the best understood of all the distance indicators.

The importance of Cepheids for distance determinations rests of course on the period-luminosity (PL) relation. The existence of this relation is well established, particularly in the Magellanic Clouds. The PL relation has relatively small scatter and the period-luminosity-colour relation seems to have negligible intrinsic scatter. A detailed discussion of some of these issues is given in Feast (1999). Furthermore it will be shown below that a reliable calibration of the Cepheid scale can now be obtained locally.

Whilst therefore we can have some confidence in the Cepheids as indicators of distance we should not be complacent. As will be indicated later there are a number of areas where an improvement of our knowledge of Cepheids is very desirable in order to further strengthen their use as distance indicators.

2. The HST Key Project Procedure

In order to focus the discussion, the present paper is limited to a discussion of factors relevant to the use which is made of Cepheids in the HST programmes for the determination of H_0 . These depend on V and I photometry. In their “final results” paper (F2001) the HST Key Project workers adopt period-luminosity relations at V and I (PL(V) and PL(I)) of the following forms;

$$M_V = -2.760 \log P - 1.458, \quad (1)$$

and

$$M_I = -2.962 \log P - 1.942. \quad (2)$$

These relations are derived from the observations of LMC Cepheids by Udalski et al. (1999, table 1), assuming the distance modulus of the LMC is 18.50. For

a Cepheid in a programme galaxy the observed (intensity weighted) mean V and I are used with these equations to derive apparent moduli. The difference between these is $E_{(V-I)}$ and multiplying this by 2.45 gives A_V and hence the true modulus.

The large number of Cepheids in the Udalski et al. work on the LMC obviously makes this data-set attractive to use as a basis. It has, however, the drawback that individual reddenings of the Cepheids themselves could not be determined (as can be done from multi-colour photometry). The reddenings were derived by dividing the LMC into a number of areas and estimating mean reddenings from the colours of giant-branch-clump stars. The mean reddening of the Cepheids used by Udalski et al. to derive equations 1 and 2 was $E_{(B-V)} = 0.147$ (not 0.10, as implied by F2001, section 3.1). The method used by F2001 only requires the relative reddenings of the LMC and their programme Cepheids. The absolute value of the LMC reddening is not required. However, F2001 have not discussed whether the Udalski et al. reddenings (which are considerably larger than have been used in the past) are consistent with their adopted true modulus of the LMC. It would also be useful in the future to have individual reddenings from multi-colour photometry (see e.g. Caldwell and Coulson 1985) for many more LMC Cepheids than are presently available. This could be used to check whether equations 1 and 2 are affected by a dependence of mean reddening on period which would affect the slope of the relations. In addition relations 1 and 2 depend heavily on short period Cepheids ($\log P \sim 0.5$) (see Udalski et al. 1999 figs 2 and 3) with very few at the long period end because of saturation of the detector used by Udalski et al. A derivation of the PL relation based on long period Cepheids would be important since the weighted mean $\log P$ of the F2001 programme Cepheids is 1.42. In addition to this there is some evidence that Cepheid moduli derived from V,I photometry require a metallicity dependent correction. This will be discussed in detail below, where it will be noted that the LMC Cepheids are metal-weak compared with the mean of the HST Key Project sample which has a mean metallicity near solar. Thus if Cepheid absolute magnitudes are based on the LMC, a metallicity dependent uncertainty will enter.

3. A Galactic Calibration of Cepheids

A calibration of Cepheid luminosities and colours based on observations in the general solar neighbourhood seems very desirable. It would avoid basing distance scales on the metal-poor LMC Cepheids. It would also be free of estimates of the LMC distance based on non-Cepheid estimators, most of which are less well understood than Cepheids themselves. However, as discussed below these non-Cepheid LMC estimates can be used together with a Galactic Cepheid calibration, to

place some limits on metallicity effects on Cepheid luminosities. Such a Galactic Cepheid calibration is now possible.

Individual reddenings of many Galactic Cepheids have been derived (Caldwell and Coulson 1987) based on the BVI system. These lead to the relation;

$$\langle V \rangle_o - \langle I \rangle_o = 0.297 \log P + 0.427 \quad (3)$$

(see Feast 1999 where a slightly more exact procedure is given). In this equation angle brackets denote intensity mean values. Using a relation of this kind to obtain reddenings, together with a PL(V) relation is equivalent to the HST Key Project procedure.

The PL(V) relation can be written;

$$M_V = \alpha \log P + \gamma. \quad (4)$$

Various estimates have been made for the slope α of this relation. Caldwell and Laney (1991) derived -2.63 ± 0.08 from SMC Cepheids; The OGLE results for the LMC discussed in the previous section gave $-2.76 \pm (0.03)$. The uncertainty is bracketed since much of the weight is in very short period Cepheids. Caldwell and Laney (1991) derived -2.81 ± 0.06 for the LMC. For Cepheids in the general solar neighbourhood Gieren et al. (1998) obtained -3.04 ± 0.14 from Baade-Wesselink luminosities. There is a slight hint here that there may be a trend of slope with metallicity since this changes in the sequence, SMC, LMC, Galaxy. It would be valuable to study this further, but at present there is no strong evidence of a significant trend of slope with metallicity. In the following we adopt for the Galaxy the result of Caldwell and Laney for the LMC, -2.81 ± 0.06 . This is the only result taken from the LMC in the present calibration.

It is of interest to see how the Cepheid distance scale would be affected by a change of slope from the adopted value, -2.81 , to the Galactic value of Gieren et al., -3.04 . The mean $\log P$ of the Cepheids used in F2001, weighted according to their contributions to the final value of H_0 , is 1.42. Most of the Cepheids used as calibrators in the discussion below are of shorter period. The weighted mean $\log P$ of the parallax calibrators is 0.8 whilst for the clusters it is 1.1. Changing the PL(V) slope from -2.81 to -3.04 would result in an increase in the parallax scale when applied to the F2001 results of 0.14 mag (a seven percent increase in distance scale) over that actually adopted. Thus if a variation in slope with metallicity in the sense tentatively suggested by the possible SMC/LMC/Galaxy trend is actually confirmed it would result in a decrease in the revised value of H_0 discussed in section 7.

A change of slope from the value adopted here (-2.81) to the LMC OGLE value (-2.76), would result in a negligibly small increase in H_0 .

There are four principal ways of obtaining a value of the PL(V) zero-point γ for galactic Cepheids (see Feast 2001).

1. A bias free analysis of the Hipparcos parallaxes of Cepheids leads to $\gamma = -1.43 \pm 0.12$ (Feast and Catchpole 1997, Feast 1999). This result and its bias free nature has been confirmed both directly and with Monte Carlo simulations (Pont 1999, Lanoix et al. 1999, Groenewegen and Oudmaijer 2000).

2. Hipparcos proper motions can be combined with radial velocities in a statistical-parallax type solution. This requires a model. The dominant effect in the case of Cepheids is that of differential galactic rotation which is clearly seen in the radial velocities and the proper motions separately. The model therefore is rather firmly based. Using this method Feast, Pont and Whitelock (1998) obtained $\gamma = -1.47 \pm 0.13$.

One can attempt a solution using the solar motion obtained from a combined discussion of solar motion and differential galactic rotation using proper motions and radial velocities. In this way the solar motion has a value which is averaged out over the whole large region covered by the Hipparcos and radial velocity Cepheids and is not confined to a small region near the Sun. The results of Feast and Whitelock (1997) imply a scale which is 0.04 ± 0.26 mag larger than that just given (Feast 2000). However, the uncertainty is too large for this solution to make any significant contribution to a final value.

The above discussion refers to the use of the systematic motions of the Cepheids. Because the velocity dispersion of Cepheids is small, any comparison of radial velocity and proper motion residuals will be sensitive to the treatment of observational scatter and probably also to group motions.

3. Pulsation parallaxes (the Baade-Wesselink method in its various forms) requires a number of assumptions to be made regarding such things as limb darkening, the colour-surface brightness relation etc. Thus whilst the internal consistency of the method is good, the external uncertainty is difficult to estimate. Feast (1999) derived, $\gamma = -1.32 \pm 0.04$ (internal) from the discussion of Laney (1998). The angular diameters of a few Cepheids have recently been determined interferometrically (Kervella et al. 2001; Nordgren et al. 2000; Armstrong et al. 2001) and the change in angular diameter of ζ Gem due to pulsation has been detected (Lane et al. 2000). When more measurements along these lines are made it should be possible to refine the pulsation parallax method further.

4. Young open clusters containing Cepheids can be used to derive γ provided the cluster scale is known. Feast (2001) shows how this scale can be rather firmly based on the well-determined Hipparcos parallax of the Hyades. This leads to $\gamma = -1.43 \pm 0.05$ (internal).

Because some of the error estimates are internal only, it is not safe to use these errors to weight the four estimates of γ given above. A straight mean gives -1.41 . The uncertainty in this value is probably somewhat less than 0.10. It should be noted that methods 1 and 2 for

determining γ lead to distance scales which do not depend on the zero-point of the reddening scale adopted, so long as consistent reddenings are applied to both calibrating and programme Cepheids. This is not the case for the other two methods. This is particularly so in the case of the clusters where the main sequence fitting depends sensitively on the adopted reddening in a way which does not cancel out in application to programme Cepheids.

4. Tests for Metallicity Effects. I

It has long been believed from observations of metal-poor Cepheids in the Magellanic Clouds that there is a metallicity effect at least in the (B-V) colours of Cepheids. Laney (quoted by Feast 1991) showed that the BVI colours of SMC Cepheids could not be brought into agreement with those in the LMC unless such an effect was assumed to exist due to the SMC Cepheids being more metal-poor than those in the LMC (a result which is known spectroscopically).

There are at least three effects attributable to metallicity differences which could affect the PL relation at a given wavelength.

1. Laney and Stobie (1986) showed from infrared photometry of galactic and Magellanic Cloud Cepheids that metallicity changes lead to a change in surface temperature at a given pulsation period.

2. There must be some effect (especially at the shorter wavelengths) due to a change of blanketing with metallicity at a given surface temperature.

Both the above have the effect of changing the bolometric corrections applicable at different wavelengths.

3. Although it is generally assumed that the bolometric PL relation is insensitive to metallicity changes, not all theorists agree on this issue.

Laney (1999) showed that the radii of Magellanic Cloud Cepheids as derived from Baade-Wesselink analyses fitted the galactic period-radius relation. Infrared photometry (Laney and Stobie 1986) shows that at a given period the metal-poor Cepheids in the Clouds are slightly hotter than the galactic ones. The evidence thus suggests that the bolometric luminosities of Cepheids of a given period increases with decreasing metallicity. However, the effect is small and within the uncertainties of the measurements.

Evidently in the case of the HST work we require to estimate the effects of metallicity changes on equations 3 and 4 above (or equivalently, equations 1 and 2). There has been some confusion in the literature since the effect on the derived distance moduli due to a metallicity change is found to be in opposite directions for the equations 3 and 4. It is thus essential to discuss the combined effect on these two equations of a change in metallicity.

The most direct empirical test of the effect of metallicity in determining distance moduli from V,I photom-

etry is that carried out by Kennicutt et al. (1998). They observed Cepheids in the galaxy M101 at different distances from the centre. At these different positions they could estimate abundances $[\text{O}/\text{H}]$ from HII region observations. Their results lead to a metallicity effect on Cepheid distance modulus determinations of $0.24 \pm 0.16 [\text{O}/\text{H}]^{-1}$ in the sense that without the correction the distance of a metal-poor Cepheid would be overestimated. This result suggests that a metallicity effect in the V,I method exists, but the uncertainty is evidently still large. (For a more detailed discussion of metallicity effects, see Feast (1999)).

5. Tests for Metallicity Effects. II. The LMC.

It has often been suggested that the Cepheid distance scale can (and should) be determined by deriving the distance to the LMC in some non-Cepheid way and then using this as the standard. However, quite apart from possible metallicity effects on the Cepheid scale, it must be born in mind that all non-Cepheid distance indicators so far used to estimate the distance of the LMC have their own problems and uncertainties. These problems include the calibration of these other indicators, their possible metallicity dependence, and, their reddenings relative to the Cepheids. Nevertheless, these non-Cepheid indicators can give a useful indication of a probable metallicity effect on Cepheid moduli.

Non-Cepheid moduli of the LMC were discussed by Feast (2001). The following is a slightly updated summary of that discussion which should be consulted for details.

1. RR Lyraes. The absolute magnitudes of RR Lyrae stars can be derived from; parallaxes (Koen and Laney 1998), parallaxes of horizontal branch stars (Gratton 1998), globular clusters with distances derived from sub-dwarf fitting (Carretta et al. 2000), δ Sct parallaxes (McNamara 1997) and statistical parallaxes (Gould and Popowski 1998). These can be combined with the data on LMC field RR Lyraes (Clementini et al. 2000) to obtain an LMC true distance modulus of 18.54.

2. Mira Variables. The infrared (K) period-luminosity relation for Miras can be calibrated using Hipparcos parallaxes (Whitelock and Feast 2000) and also using Miras in globular clusters with cluster distances on the subdwarf scale of Carretta et al. (2000). Using these calibrations with the LMC Mira data (Feast et al. 1989) gives (Feast, Whitelock and Menzies, to be published) an LMC modulus of 18.60.

3. The ring round SN1987A. The best estimate of the LMC modulus from this is probably that of Panagia (1998) which is 18.58.

4. LMC globular clusters. Main-sequence fitting (Johnson et al. 1999) of LMC clusters can be used to derive a modulus of 18.52.

5. The red giant clump. The use of the red giant clump as an LMC distance indicator is complicated by age and metallicity effects and by reddening uncertainties. The best estimate is probably that of Girardi and Salaris (2001) who find a modulus of 18.55.

6. Eclipsing Variables. Although the use of eclipsing variables as distance indicators seems rather straight forward in theory, its application to the LMC requires at present a number of assumptions. The best current estimated of the modulus by this method is probably that of Groenewegen and Salaris (2001) who obtain 18.42.

The real uncertainties of the above estimates are probably about 0.1mag. The first three estimates seem likely to be the most secure and a straight mean of them is 18.57. A straight mean of all six estimates is 18.54. The V,I distance modulus of the LMC using equations 3 and 4 and $\gamma = -1.41$ is 18.66, uncorrected for metallicity effects. If we adopt a metallicity effect of $0.2 \text{ mag } [\text{O}/\text{H}]^{-1}$ and $[\text{O}/\text{H}]_{\text{LMC}} = -0.4$ as is done by F2001 following Kennicutt et al. (1998), we obtain a corrected Cepheid modulus of 18.58. The close agreement of this value with the mean non-Cepheid estimate is no doubt partly fortuitous since the uncertainties in both estimates are probably of order 0.1. However, the results do suggest that a metallicity correction of the approximate amount suggested by Kennicutt et al. is present in V,I estimates.

6. A Cepheid test using NGC4258

A distance to the galaxy NGC4258 has been derived from the motion of H_2O masers in the central region and a simple model (Hernstein et al. 1999). Newman et al. (2001) have recently published HST V,I observations of Cepheids in this galaxy which can thus also be used to derive a distance. The metallicity adopted by Newman et al. (from HII region measurements by Zaritsky et al. 1994) is slightly below solar ($[\text{O}/\text{H}] = -0.05$). Thus any reasonable metallicity correction to a galactic calibration will be very small. Adopting the galactic calibration given above and a metallicity effect of $0.20 \text{ mag } [\text{O}/\text{H}]^{-1}$ one obtains a true distance modulus of 29.53 ± 0.17 , where the standard error is taken from the discussion of Newman et al. together with an estimated error of the galactic zero-point of ~ 0.10 . In deriving this value we have followed the procedure of Newman et al. and used template-fitted DoPHOT mean magnitudes kindly supplied by Dr Newman. These magnitudes differ slightly from the values given in Table 2 of Newman et al. A metallicity correction of -0.01 mag has been applied. The Cepheid distance is therefore greater than the maser one by $0.24 \pm 0.21 \text{ mag}$. This difference is not significant.

7. Key Project Value of H_0 based on a Galactic Calibration

F2001 have summarized the HST Cepheid Key Project data together with related HST data by other groups (especially the SNIa group). As indicated in the Introduction they use an adopted LMC modulus as the basis of their analysis. They also introduce a metallicity correction for the first time in their series of papers. They derive a value of $H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Using their data and with the same metallicity term ($0.2 \text{ mag [O/H]}^{-1}$) but with the galactic calibration derived above, one obtains $H_0 = 67 \pm 8$ where the standard error is taken from F2001. These two estimates of H_0 are gratifyingly close. Amongst the reasons for preferring a scale based on the galactic calibration is that it is practically immune to the uncertain metallicity correction. This follows since the mean metallicity of the F2001 Cepheid galaxies, weighted according to their contribution to the final value of H_0 is close to solar ($[\text{O}/\text{H}] = -0.08$). It has been hypothesized that the metallicity correction could be as high as $0.6 \text{ mag [O/H]}^{-1}$. Whilst it seems unlikely that it could be as large as this, even such a large value will have only a very small effect on H_0 derived using the galactic calibration. On the other hand, since $[\text{O}/\text{H}]_{\text{LMC}} = -0.4$, such a large correction coefficient would have a significant effect (a six percent decrease in H_0) on a calibration based on an adopted LMC modulus.

It should be made clear that the above discussion is given to illustrate how the galactic calibration affects the conclusions of F2001. There appears still to be considerable differences in the interpretation of the HST data by different groups, unrelated to the adopted basic Cepheid distance scale (see for instance Saha et al. 1999). Full agreement on these matter is required before the value of H_0 can be considered properly established.

8. Conclusions

The calibration of the Cepheid zero-point is now quite well established from galactic observations. The present uncertainty is about 0.10 mag . It is obviously desirable to improve this accuracy, though it is not clear that this can be achieved without further astrometry from space (GAIA etc.). It is evident that further work is needed on the PL(V) and PL(V-I) slopes at long periods and it would be very desirable to determine empirically if these slopes depend on metallicity. In doing this it would be essential to derive individual reddenings for the Cepheids from multicolour photometry. It would also be of considerable interest to measure the metallicities of many more galactic Cepheids. In particular it would be useful to know more precisely the spread in metallicities amongst local Cepheids.

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